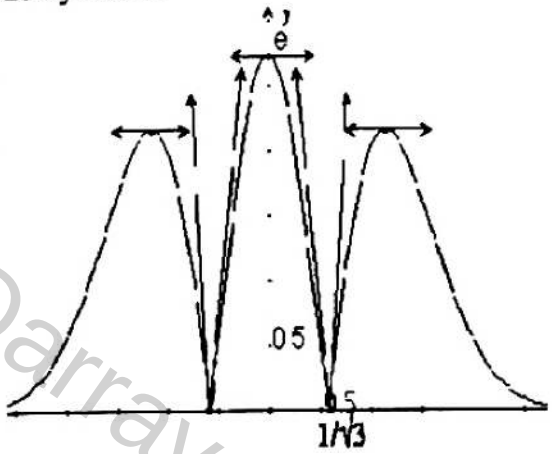
x	0	$\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	$+\infty$
$f'(x)$	$-$	$\|\|$	$+\ 0\ -$	
$f(x)$	e ↘	0 ↗	$3e^{-\frac{1}{3}}$ ↘	0

6) $y = 0$ asymptote à Cf et $(O\vec{j})$ axe de symétrie



EX21

$n \in IN$

$I_n = \int_0^{\frac{\pi}{2}} e^{-nx}.\sin x.dx$

$J_n = \int_0^{\frac{\pi}{2}} e^{-nx}.\cos x.dx$

1) * $I_0 = \int_0^{\frac{\pi}{2}} .\sin x.dx = [-\cos x.]_0^{\frac{\pi}{2}} = 1$

$J_0 = \int_0^{\frac{\pi}{2}} .\cos x.dx = [\sin x.]_0^{\frac{\pi}{2}} = 1$

2) $n \in IN^*$

* $I_n = \int_0^{\frac{\pi}{2}} e^{-nx}.\sin x.dx$

$\begin{cases} u(x) = e^{-nx} \rightarrow u'(x) = -n\ e^{-x} \\ v'(x) = \sin(x) \rightarrow v(x) = -\cos(x) \end{cases}$

$I_n = [-e^{-nx}\cos x]_0^{\frac{\pi}{2}} - n\int_0^{\frac{\pi}{2}} e^{-nx}.\cos x.dx$

$I_n = 1 - nJ_n$, d'ou $\boxed{I_n + nJ_n = 1}$ (1)

* $J_n = \int_0^{\frac{\pi}{2}} e^{-nx}.\cos x.dx$

Soit

$\begin{cases} u(x) = e^{-nx} \rightarrow u'(x) = -n\ e^{-x} \\ v'(x) = \cos x \rightarrow v(x) = \sin x \end{cases}$

$J_n = [e^{-nx}\sin x.]_0^{\frac{\pi}{2}} + n\int_0^{\frac{\pi}{2}} e^{-nx}.\sin x.dx$

$J_n = e^{-n\frac{\pi}{2}} + nI_n$

d'ou $\boxed{J_n - nI_n = e^{-n\frac{\pi}{2}}}$ (2)

(1) et (2) donne

$\begin{cases} I_n + nJ_n = 1 \\ -nI_n + J_n = e^{-n\frac{\pi}{2}} \end{cases}$

3)

$\begin{cases} I_n + nJ_n = 1 \\ -nI_n + J_n = e^{-n\frac{\pi}{2}} \end{cases}$

$\Leftrightarrow$

$\begin{cases} nI_n + n^2 J_n = n \\ -nI_n + J_n = e^{-n\frac{\pi}{2}} \end{cases}$

$(1) + (2) \Rightarrow (1+n^2)J_n = n + e^{-n\frac{\pi}{2}}$

$\Rightarrow \boxed{J_n = \frac{n + e^{-n\frac{\pi}{2}}}{n^2+1}}$

Qarray.tn -Aqra Blech- Qarray.tn